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A Statistical Analysis of the Role of Critical-Thinking Games in College Students' Quantitative Reasoning

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1 Abstract

Improving college students' quantitative reasoning is crucial to increase STEM-field retention rates. The National Survey of Student Engagement (NSSE), results indicate that "college students don't always get exposure to activities that develop [quantitative reasoning]" (Berrett & Sander, 2013). This study aimed to engage college students in quantitative reasoning through critical-thinking games. Phase 1, conducted in summer 2014 with a university research grant, involved a correlation analysis of the work of 133 college students enrolled in Elementary Statistics I, Chemical Principles II, Elements of Calculus, Pre-Calculus with Trigonometry, Multivariable Calculus, and Differential Equations. Participants held a variety of 39 different majors, with each college at the university represented. Results showed a positive correlation between students' performance in games involving critical-thinking skills and their performance in a mathematics-based course. The strongest correlation was between a card game's scores and mathematics-based course grades ($r=0.549$). Phase 2, extends the research with a more focused study on the use of this card game in regard to math performance. This phase includes 18 students, all of whom are taking the same course, Calculus I, with the same professor, in order to eliminate extraneous variables. The students are taking a pre and post logic-test and playing ten minutes of the card game once a week. Data will be gathered until November 2014 and will be analyzed by Hypothesis Testing using computer analysis software. The research also includes analysis of patterns of error that correspond to the eight standards for mathematical practice outlined in the common core.

2 Introduction

Psychologists have known that everyone is born with an “approximate number sense, called ANS" for quite some time now (Hyde, 94). ANS is the idea that children are born with the ability to estimate the amount of something, without having to count it. However, a recent study at the University of Illinois at Urbana-Champaign has shed light on the results of engaging peoples' ANS. This study has concluded that by exercising a person's ANS you can better their mathematics performance. (Hyde, 94) How to engage and exercise this sense that everyone is born with is a hard, but a rewarding feat in mathematics.

Phase one of this research will test the hypothesis: if there is a correlation between mathematical game scores and students performance in mathematics classes. The correlation coefficient will determine whether or not games involving strategy, logic, and mathematical reasoning are performed well if and only if the student is strong in mathematics. The correlation coefficient will determine if there is a connection between the logical thinking that most of the games require and the ability for students to perform well in their mathematics classes. Determining this correlation will help to open the doors to new strategies that could be incorporated in mathematics education. Games are motivational and engaging tools, but their value to a college student's education is unknown. The results of this research will help to determine if mathematical games can help to foster mathematical reasoning and problem solving skills. Looking at games that strictly have critical thinking skills is a new way to approach the way that students grow into more dominant mathematicians.

Development of students’ quantitative reasoning is also important to mathematical success. Quantitative reasoning (QR) is an important skill for students that should be obtained by the time

that the exit their undergraduate career. QR focuses on student's ability to reason in a context outside of the classroom. One of the many goals of quantitative reasoning is giving students the ability to be competent in pattern recognitions (Hurney, 2011). The National Survey of Student Engagement (NSSE), results indicate that "college students don't always get exposure to activities that develop [quantitative reasoning]" (Berrett & Sander, 2013). Phase 2 of this research will also analyze if there is a correlation between playing games and an improvement in mathematical logic puzzles. Providing evidence to whether or not these games would be appropriate replacements to homework. As an educational tool, games may not be considered as cumbersome as homework or studying. Games involving problem solving skills that are not strictly "mathematical" games are more likely to be played by students' on their own time. If there is a positive relationship, this will be a very important learning tool that educators could use in the classroom. This strategy will help to cultivate students' abilities in one of the most important and useful aspects of Mathematics, problem solving.

3 Methodology

3.1 Phase One

3.1.1 Introduction

The goal of phase one was to determine if there was a correlation between students' performance in college mathematics based courses and their performance in games involving mathematics reasoning. All participants played two games at the conclusion of a college mathematics course. Participants also completed a survey that consisted of some basic demographic information and their scores from the games that were played. The scores of both of these games where analyzed against a GPA equivalent of each participants grade in the corresponding mathematics class.

3.1.2 Participants

Participants in phase one consisted of 133 Bridgewater State University students. The participants in his group held 39 varying majors with each college at the university being represented. Each participant took a mathematics based course during the spring, summer, or fall semester of 2014. The mathematics based courses included; Multivariable Calculus, Elementary Statistics I, Chemical Principles II, Differential Equations, Elements of Calculus I, Pre-Calculus, and Quantitative Method Management. The participants were separated based on the level of the mathematics based course that they were completing at the time of the survey. Participants labeled under the 100 level cohort consisted of students that took either Elementary Statistics I, Chemical Principles II, Elements of Calculus I, Pre-Calculus, or Quantitative Method Management at the time of the survey. Participants labeled under the 300 level cohort where students of either Differential Equations or Multivariable Calculus during the time of the survey.

3.1.3 Game Choice Reasoning

All participants played two games, SET® and an online game applet. All games involved, critical thinking and any math skill required was taken into consideration. SET®, the family game of visual perception, is a card game (figure 3). In order to actively engage all participants, math inclined or not, I chose this game since it contains no numbers. This ensured that my participants would not think of this game as a mathematical game and would not link any predetermined dispositions of their math performance with the game. The cards consist of only colored shapes, which participants must arrange into sets of three cards following specific rules as to the characteristics of each card. The 1st round of the game is challenging, but as the rules are learned the game becomes easier. For this reason I chose to eliminate any students from the

study who had ever played the game before. While proctoring the game play, each class was given the same word-for-word instructions of the game. This ensured that each student, from each class, was at an equivalent level when beginning the game. The students were placed in groups of 3 or 4 and played the game for 12 minutes.

In addition to the SET® game, each cohort was assigned a corresponding online game. These online games contained more math content; however the level of the content was appropriate for each cohort. The 300 level students were assigned a vector game[1] (figure 1) and the 100 level students were assigned a coordinate game[2] (figure 2). Vectors is a topic that is covered heavily in Calculus III, which is why only participants in Differential Equations and Calculus III where given this game. The vector game involved adding and subtracting vectors to move up steps that navigated from a starting position to a final endpoint. The game involves 8 multiple choice questions, with an unlimited amount of guesses on each question. After completing the game the user is presented with a score that indicates how many wrong answers and how many correct answers they gave. In order for this game to be completed, the number of correct answers must only equal eight and the participants were scored based on how many wrong answers that they gave (0-24).

The coordinate game only required basic knowledge of plotting (x, y) coordinates in the first quadrant. The object of the game was to maneuver from the origin of the coordinate plane to the location (20, 20) without hitting any mines. The game was set to have 15 random mines on the board each game. Participants were instructed to continuously play the game for 10 minutes trying to complete each game as quickly as possible and with as little moves as possible. Each participant then recorded their score which indicated the number of completed problems, the total number of moves, the number of times destroyed (tallied each time they ran into a mine),

and the number of incomplete problems. The final numerical score for this game was calculated by their number of moves per game, with their amount of times destroyed calculating into their number of moves. Their score was calculated by adding together the number of total moves with the number of times destroyed and dividing that by the number of games that each student played ($score = \frac{total\ moves + times\ destroyed}{number\ of\ games}$).

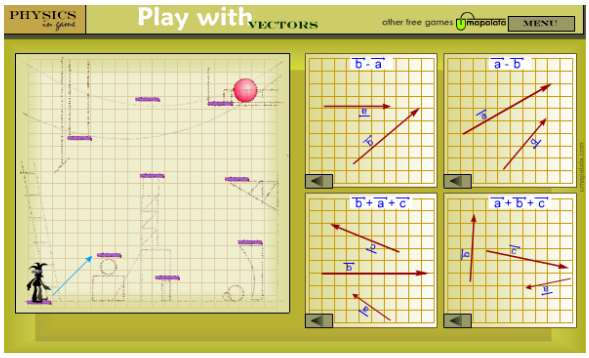


Figure 1

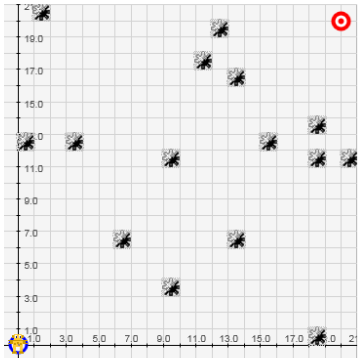


Figure 2

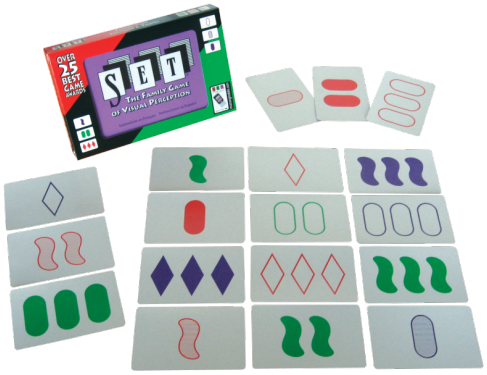


Figure 3

3.2 Phase Two

3.2.1 Introduction

Phase two, is an extension of the data gathered in phase one. Phase one led us to wonder if playing these games regularly would be correlated with an improvement in critical thinking

performance. Phase two focused on determining if students change in performance of a logic test was correlated with their participation of a critical thinking game. The game that these participants played was determined by the game that showed the strongest correlation in phase one of this study (SET®). This phase involved 18 Bridgewater State University students, all taking the same Calculus I class in the fall 2014 semester. Each participant took a pre-test as well as a post test. The test was a logic based test that was adapted from a Kangaroo Math Contest practice test. The questions were designed to simulate problem solving skills, without requiring any higher level arithmetic or other mathematical skills. The pre-test (see 7.3) and post-test (see 7.4) consisted of the same questions, with altered answers. Participants where categorized into two groups, 9 participants in each a variable and control group. The control group took the pre-test and 5 weeks later completed the post-test, no extra participation was required for this group. The other group, played ten minutes of a game every week for 5 weeks in between the pre and post-test.

3.3 Correlation and Linear Regression

To determine if there is a correlation between students' performance in games and their corresponding math grades, a correlation analysis is used. Correlation is a computation that is designed to study relationships and association between variables. The *correlation coefficient* , *r,* is used is used to quantify and represent the relationship and it can tell us whether the correlation is low, moderate, or high. The coefficient, *r*, can range from -1 to +1; $r = -1$ indicates a perfect negative correlation, $r = +1$ indicates a perfect positive correlation, and $r = 0$ indicates no correlation is present between the variables. The value of *r* is given by:

$$r = \frac{n\sum xy - (\sum x)(\sum y)}{\sqrt{n\sum x^2 - (\sum x)^2}\sqrt{n\sum y^2 - (\sum y)^2}}$$

where *n* represents the sample size. When interpreting a correlation coefficient, it is important to note that the strength of the correlation is determined by the absolute value of r. A correlation coefficient from .00 to .20 is considered low to no correlation, .20 to .40 is a low correlation, .40 to .60 is moderate, .60 to .80 is a high correlation and .80 to 1.00 is substantial.

Once a correlation is determined, *regression* is used in order to make predictions following the means of the study. When one variable is used to predict another, it is called simple regression. If the data indicates a linear relationship between the independent and dependent variable, the least-squares criterion can be used to find the equation of the best-fitting line. The Least-Squares line, is in the form of $\hat{y} = a + bx$, where *x* is the independent variable, *y* is the dependent variable, *a* is the intercept and *b* is the slope. The *regression coefficients, a* and *b,* are calculated through the following equations:

$$b = \frac{n\sum xy - (\sum x)(\sum y)}{n\sum x^2 - (\sum x)^2}$$

$$a = \bar{y} - b\bar{x}$$

Once the least squares line is found, the equation can be used to evaluate predictions, $\hat{y}$, for any *x* in the domain of the problem. It is important to note that correlation does not imply causation and that predictions do not always transpire.

3.4 Hypothesis Testing

To check if the two variables are related, a hypothesis test is performed. Hypothesis testing poses two situations, the null hypothesis and the alternate hypothesis. The null hypothesis, denoted H_o,

describes either a situation that is already known or that there is no relationship between two variables. The alternate hypothesis, denoted H_1, is usually the negation of the null hypothesis, or what the data is testing. In order to reject a null hypothesis, the level of significance, α, must be determined. Once α is determined, it is compared to the *p*-value, which indicates the probability that the test statistics will fall under the conditions of the null hypothesis. If the *p*-value $\leq \alpha$ the null hypothesis is rejected and there is enough evidence to state the alternate hypothesis.

To analyze the data collected in phase one of the study, a correlation analysis will be used. The correlation coefficient found will determine whether or not there is enough evidence to reject the null hypothesis. The null hypothesis is defined as, H_o: $\rho = 0$, or that there is no correlation. I am testing the alternate hypothesis that, $H_1 : \rho \neq 0$, that there is a correlation and it is either positive or negative. For this test, α= 0.05 and the values of p necessary to reject the null hypothesis, are represented by values at are less than or equal to 0.05.

Phase two will be an analysis of the mean of two data sets; a test group and a control group. The data sets will include the change in scores from a pre-test to a post-test. For phase two the null hypothesis is H_o: $\mu \leq 0$, in other words, the average change in scores indicates that participants remain the same or decrease in scores from the pre-test to the post-test. The alternate hypothesis, the hypothesis that I am testing, is denoted by $H_1 : \mu < 0$. The alternate hypothesis that is being tested will indicate that the average change in the participants' scores are positive, meaning they had an increase in performance. Phase two will aim at determining the hypothesis that the control group will fall under the null hypothesis and the test group will fall under the alternate hypothesis. Due to a low number of participants (n = 18) a t-Test must be completed in order to test the significance of the results. From the Table of Critical Values for t-Tests the critical value

at a significance of 0.05 with 17 degrees of freedom is 2.11. This indicates that the t value found by the formula:

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

Where: $\bar{X}_1$ = Mean of the test group

$\bar{X}_2$ = Mean of the control group

S_1^2 = Variance of the test group

S_2^2 = Variance of the control group

n_1 = Number of people in the test group

n_2 = Number of people in the control group

If the calculated value of *t* is less than the critical value of 2.11 then the results are statistically significant at the $p<0.05$ level.

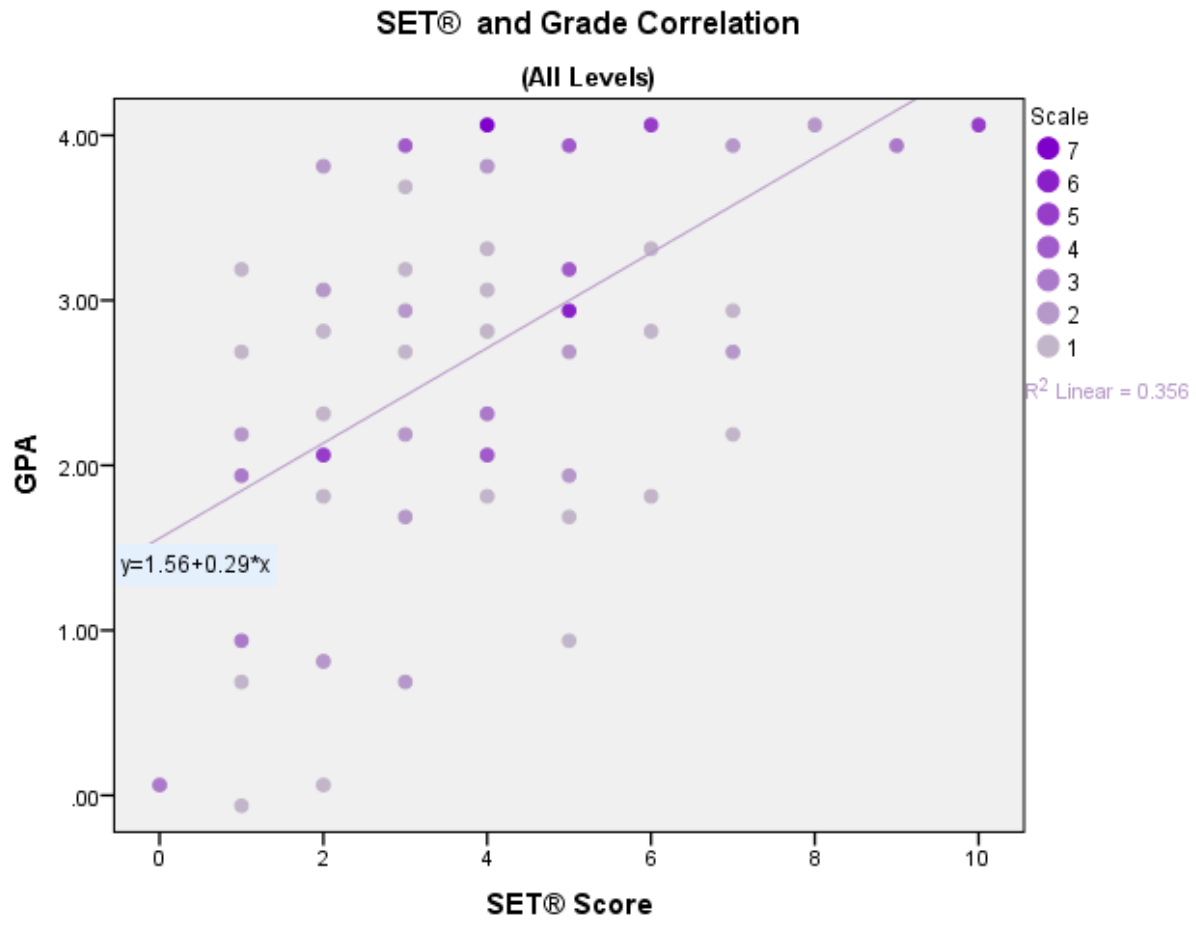
4 Results

4.1 Phase One

Each of the students' scores in each of the games that they participated in was compared to their grade from their current class in order to determine if any correlation exists. It is important to note that there is both a limited domain and range in all of these situations. Since the range is the GPA scale, the range = {0.00, 0.7 ,1.0 ,1.3 ,1.7 ,2.0 ,2.3 ,2.7 ,3.0 ,3.3 ,3.7 ,4.0} and the domains vary in accordance to how each game is scored. The SET ® game is scored from 0 to 27, the

vector game is scored from 0 to 24, and the coordinate game is score starts at 0, but has no maximum. Due to the limited range and domains of these functions, correlation may not be visually obvious. However it is also important to note that due to the limited domain and range values in this study, there are many repeated data points. All of the graphs indicate a scale that helps emphasize these points. In the following graphs the shading of each data point has been adjusted to represent repeated data values. The darker that the data point appears, the more data points it represents. Figure 4 shows the correlation that exists between every participant's (both 100 and 300 level cohorts) SET score and a GPA equivalent of the grade they received in the corresponding class.

Figure 4



The correlation defined in this graph is defined with the correlation coefficient of r = 0.5967 and the correlation is significant at the 0.01 level. Although the SET game was played by all participants, figures 5 (100-level) and 6 (300-level) separates them to analyze their correlation separately, to determine any trends that may emerge from different level students.

Figure 5

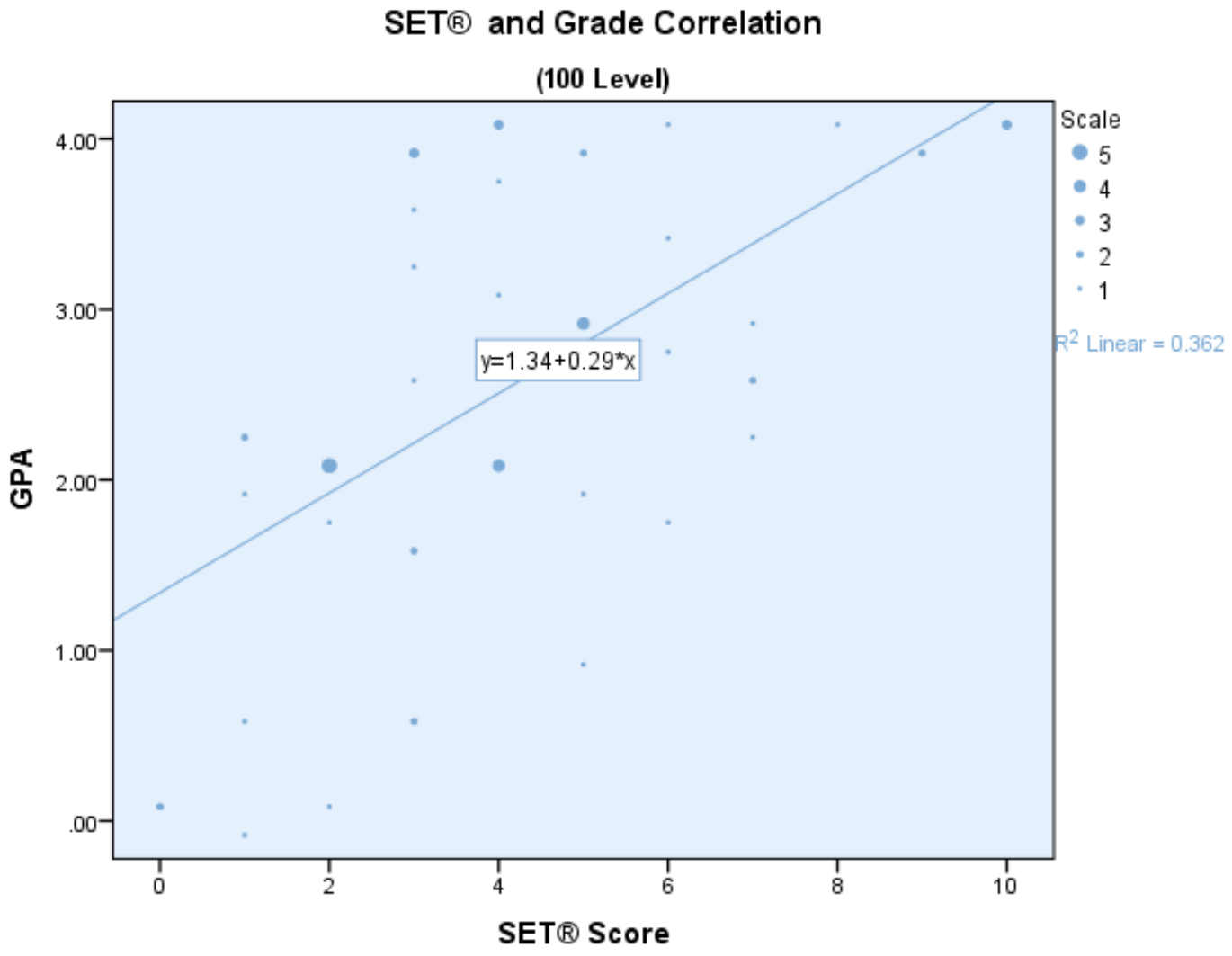
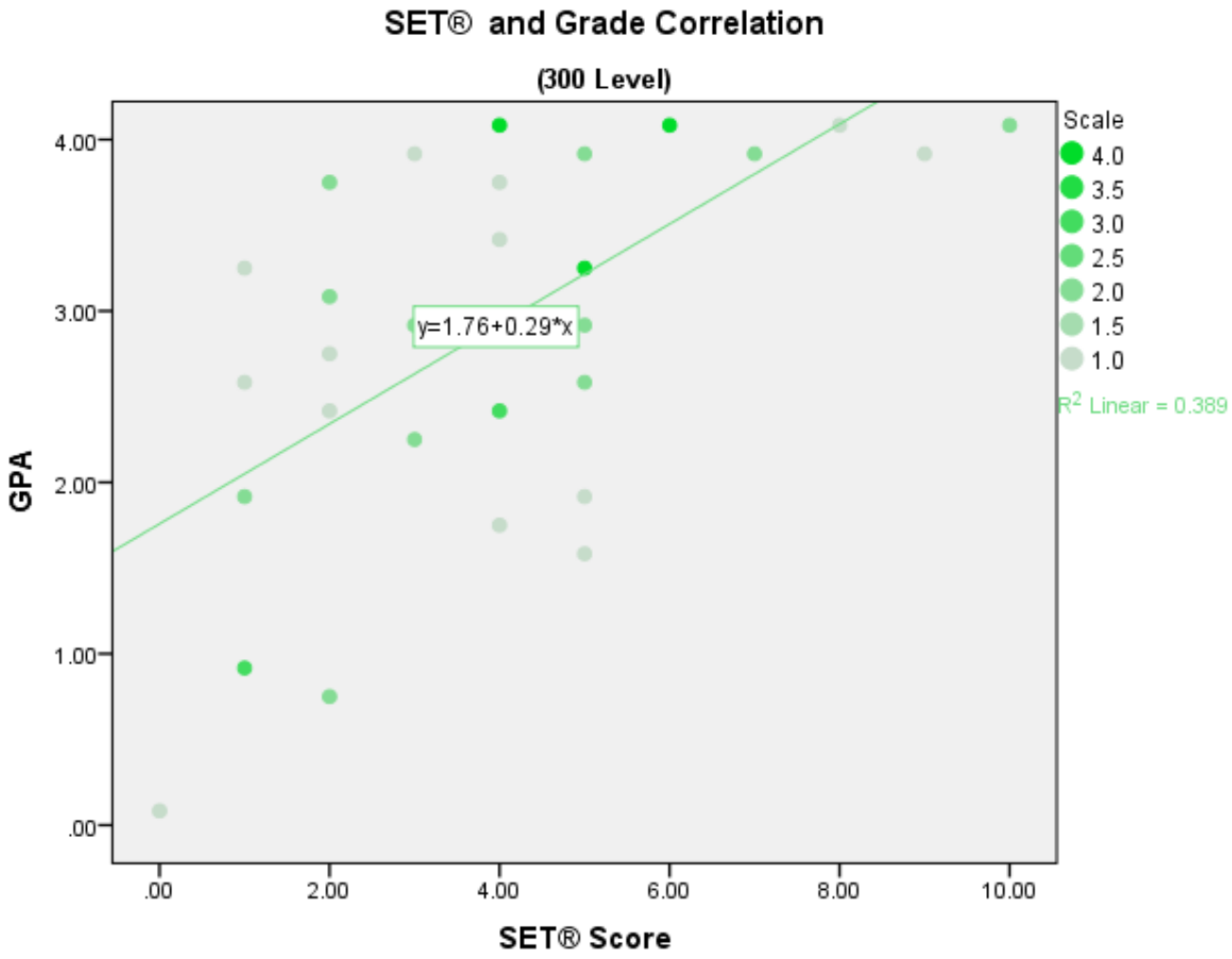
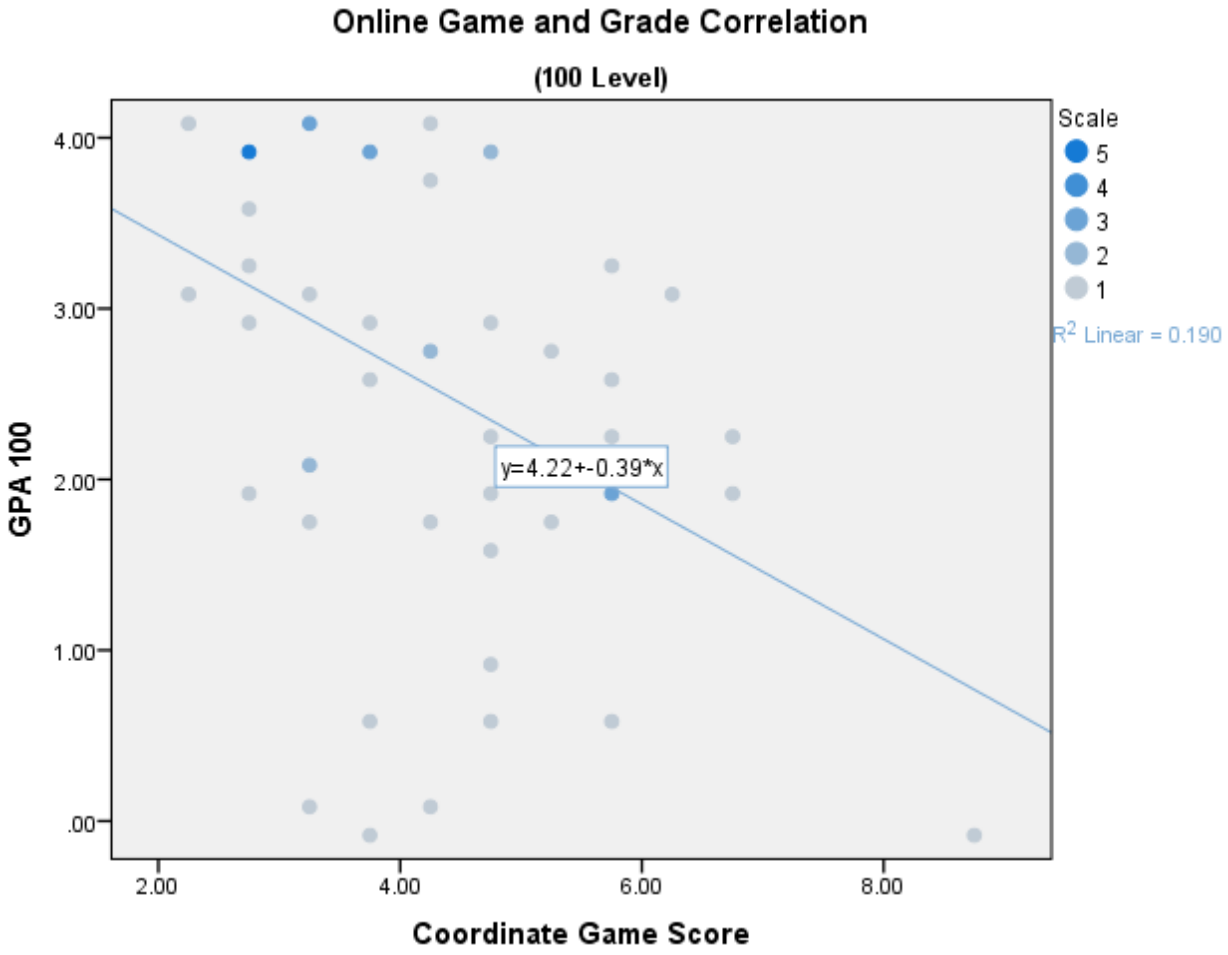


Figure 6



The 300 level students have a slightly stronger correlation with $r = 0.624$ and $p < 0.01$ than the 100 level students with $r = 0.602$ and $p < 0.01$. The online games that the participants played are separated by level, since each level played a game tailored to their class level. Figure 7 represents each students score in the online vector applet that is played by students in the 100 level classes and each student's corresponding grade.

Figure 7



The graph shows that the students grade and their scores from the online game are negatively correlated with r = -0.4359 and p <0.01. The negative correlation is expected due to the nature of the online applet score. The score represents moves per game, since the participants were instructed to complete as many games as possible in as few moves as possible a low score is considered a better score than a high score. The online game that was used in the 300 level classes will also show a negative correlation (figure 8) since this online game also reflects a lower score as a better score. The vector game was scored based on how many wrong answers the participant entered, if they had entered no wrong answers their score would be a zero. Also note that the most wrong answers a participant can enter is 24.

Figure 8

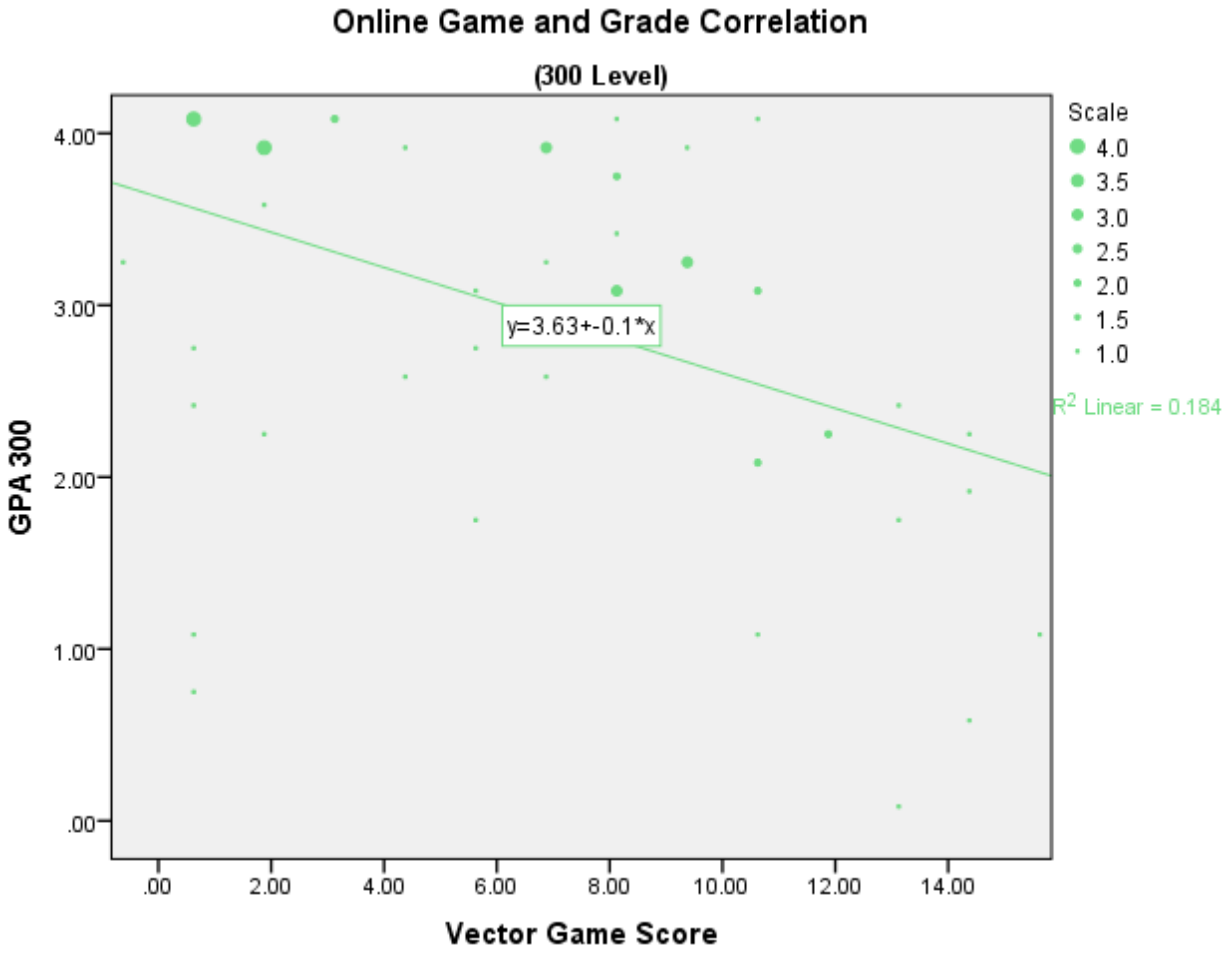


Figure 8 shows the correlation of the results of the 300 level online game and the grades in the corresponding classes. The correlation that is represented by these data points has the correlation coefficient of $r = -0.429$ and $p < 0.01$.

4.2 Phase Two

Due to the results of phase one, the participants in phase two played the SET® game for 10 minutes a week for the 5 weeks between the pre-test and post-test. The pre-test and post-test where both graded based on correctness. The participants were instructed not to guess an answer

on the test if they didn't know the answer. Each question received points, 1 point for every correct answer, 0 points for every unanswered question and -1 points for ever incorrect answer. Since both tests where 5 questions long, the scores can range from -5 to 5. The following table shows the change in each of the 18 participants test scores (post-test score – pre-test score).

Change in Test Group Scores		Change in Control Group Scores
+2		-2
0		0
+4		0
+2		-2
0		+3[1]
-1[1]		-1
+2		0
+2		-1
0		0
Mean	+ 1.50	−0. 5
Standard Deviation	1.41421	0.57735
T-Test	0.001902	

It is important to note that a zero indicates no change in score, a positive number indicates an improvement and a negative indicates a decrease in performance.

[1] These two outliers were eliminated before any statistical tests were completed.

5 Discussion

5.2 Phase One

With a significance of $p = 0.05$, there is enough evidence to state that there is a positive correlation between students' performance in mathematics and their performance in games involving critical thinking skills and to reject the null hypothesis (H_o: $\rho = 0$) Between the three games presented, the SET® game shows a moderate to high correlation, with a correlation coefficient of $r= 0.5967$ and $p = 0.00089$ ($p < \alpha$). The same trend is apparent among the other two games, validating that this is not only conclusive with the SET® game.

The results of phase one indicate that you may be able to predict a students' grade by their performance in the game of SET®. This game involves critical thinking and problem solving skills that are essential for students to succeed in math classes. From this research, SET would best be used in a classroom to "pre-test" a student on their critical thinking and problem solving skills without making them perform word problems. From looking at the equations of the regression lines formed from each of the games, it is apparent that a student's score may not be predicted from the playing of these games. Just from looking at the equations that emerged from separating the SET® game into the two cohorts, it is apparent (from the *y*-intercepts) that these results cannot be used to predict grades in other classrooms. The 100- Level cohort's regression line with respect to the SET® game was defined by $y = 1.34+0.29x$ and the 300-Level cohorts regression line with respect to the SET® game was defined by $y =1.76 +0.29x$. Although the use of this game could be used as a pre-test, it would be better used for ranking students, rather

than predicting their grades. These results only reflect students who have never played the game of SET® before, and may not be as effective if the student has played before.

5.2 Phase Two

Phase two involved the use of the SET® game since it was the game that showed the highest correlation in phase one. This game is the only game that can produce an accurate score since it eliminates aspects of luck that would occur when students are guessing. While playing the vector game, students may guess randomly until they finish, likewise with the coordinate game. Since the SET® game eliminates any factors of luck; it is the only game that can provide honest scoring. The game was played every week for ten minutes by every participant of the test group. Although it does not affect the results, their scores in the SET game where recorded and did increase each time they played the game. This gave more validity to the fact that the correlation from phase two may not apply to people who have played the game before.

In the test group, all but one participant improved or maintained the same score from the pre-test to the post-test. In the control group, all but one participant declined or maintained the same performance between tests. After performing a t-Test I found that t was valued at *0.001902* and less than the critical value of 2.11. This provides us with enough evidence to support the alternate hypothesis, that $\mu \geq 0$ or the average change in the participants' scores are positive, meaning they had an increase in performance at a 0.05 significance level.

Each of the logic based test questions were aligned to at least one of the eight standards for mathematical practice that are outlined in the common core:

- CCSS.Math.Practice.MP1: Make sense of problems and persevere in solving them.

- CCSS.Math.Practice.MP2: Reason abstractly and quantitatively.
- CCSS.Math.Practice.MP3: Construct viable arguments and critique the reasoning of others.
- CCSS.Math.Practice.MP4: Model with mathematics.
- CCSS.Math.Practice.MP5: Use appropriate tools strategically.
- CCSS.Math.Practice.MP6: Attend to precision.
- CCSS.Math.Practice.MP7: Look for and make use of structure.
- CCSS.Math.Practice.MP8: Look for and express regularity in repeated reasoning.

The most significant trend that emerged from this analysis was between questions 1 and 5 (Refer to 7.4 and 7.5 for pre-test and post-test questions). These two questions were a lined with standards CCSS.Math.Practice.MP7 and CCSS.Math.Practice.MP8. On the pre-test, every participant who answered number 1 incorrectly also answered number 5 incorrectly. This trend demonstrates that participants in this study have an apparent weakness in both of these standards. Following these two questions to the post-test, it was found that the every participant in the test group who had previously answered question 1 incorrectly had now correctly answered the question. Some improvement was shown in question 5, however not as apparent or as vast as the change in number 5 (the level of difficulty in these two questions is very different). Since there was no such pattern apparent in post-test between question 1 and 5, there seems to be no universal rate at which these standards are met.

The results, from this sample, indicate that the game did correlate with an improvement in the student's ability to think critically. These results may lead educators to believe that the critical thinking skills are more readily built than arithmetic skills, and that it is the combination of both of these aspects that causes students to succeed in mathematics. Although playing these

games may not be linked as a cause for improvement in mathematics, it will build critical thinking skills, which is an important key to excelling in mathematics.

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Links to Games:

1. http://www.cut-the-knot.org/Games/Vectors.shtml
2. http://www.shodor.org/interactivate/activities/SimpleMazeGame/

Pretest Adaptation:

http://www.mathkangaroo.org/mk/default.html

7 Research Materials

7.1 Phase 1 Survey – 300 Level

"Math Games" Survey and Consent Form **ID Number: _____**

If you are at least 18 years of age, you are invited to participate in this 30 minute study about math games. Although you may not personally benefit, this study is important to learn whether critical thinking skills may be influenced by mathematics games. There are no foreseeable risks, your responses will be held in confidence, and you may refuse to answer particular questions or withdraw from the study at any time. If you have no further questions and agree to participate, please continue.

What is your current major(s): _____

In the last month, how often have you played any type of game? (board, card, apps etc.)

1	2	3	4	5
(once a month or less)	(2-3 times a month)	(Once a week)	(2-5 times a week)	(at least once a day)

What game (or game type) do you play most often: _____

(Write none if not applicable)

SET:

Have you ever played the game of SET before today? Yes No

What was the total number of points that you earned in the game SET: _____

(note: one point is received for every set or every three cards you obtained)

Online Game: Play the online applet **twice** and then record your score from your **second attempt.**

Have you ever played the vector online applet before today? Yes No

What score did you receive in the vector applet game?

_____ _____

(Score in the green box) *(Score in the red box)*

7.2 Phase 1 Survey – 100 Level

"Math Games" Survey and Consent Form **ID Number: _____**

If you are at least 18 years of age, you are invited to participate in this 30 minute study about math games. Although you may not personally benefit, this study is important to learn whether critical thinking skills may be influenced by mathematics games. There are no foreseeable risks, your responses will be held in confidence, and you may refuse to answer particular questions or withdraw from the study at any time. If you have no further questions and agree to participate, please continue.

What is your current major(s): _____

In the last month, how often have you played any type of game? (board, card, apps etc.)

1	2	3	4	5
(once a month or less)	(2-3 times a month)	(Once a week)	(2-5 times a week)	(at least once a day)

What game (or game type) do you play most often: _____

(Write none if not applicable)

SET:

Have you ever played the game of SET before today? Yes No

What was the total number of points that you earned in the game SET: _____

(note: one point is received for every set or every three cards you obtained)

Online Game: Play the game and record your score at the end.

Have you ever played this online applet before today? Yes No

Complete the following by clicking on the "**Show Score**" button

Completed Problems : _____

Total Moves: _____

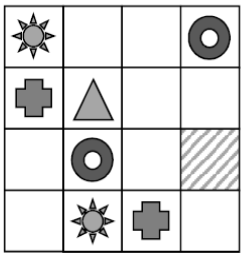
Total Times Destroyed: _____

Incomplete Problems: _____

7.3 Phase 2 Pre-Test

If you are at least 18 years of age, you are invited to participate in this 10 minute study about math games. Although you may not personally benefit, this study is important to learn whether critical thinking skills may be influenced by mathematics games. There are no foreseeable risks, your responses will be held in confidence, and you may refuse to answer particular questions or withdraw from the study at any time. If you have no further questions and agree to participate, please continue.

1 Alice has to place different shapes in the table so that each distinct shape appears exactly once in each row and each column.



What shape should Alice place in the grey square?

(A)  (B)  (C)  (D)  (E) 

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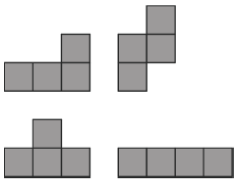
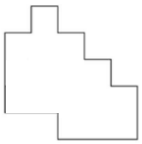
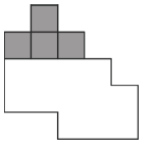
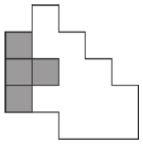
2 Ann has four pieces as shown.

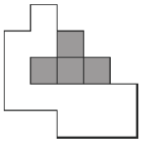


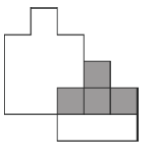
With these pieces she can completely cover the shape:  .

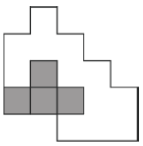
Where should she put the piece  ?

(A) 

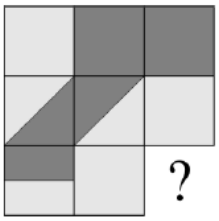
(B) 

(C) 

(D) 

(E) 

3 Which tile must be added to the picture so that the total light area is as large as the total dark area?



(A)  (B)  (C)  (D)  (E) 

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4 Mary had equal numbers of white, black and striped tokens. She used some of the tokens to make the pile shown in the figure.

She still has five tokens which are not in the pile. How many black tokens did she have in total?

(A) 5 (B) 6 (C) 7 (D) 10 (E) 15

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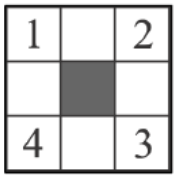
5 Nick has written each of the numbers from 1 to 9 in the cells of the 3 × 3 table. Only four of these numbers can be seen in the figure. Nick has noticed that for the number 5, the sum of the numbers in the neighbouring cells equals 13 (neighbouring cells are cells sharing a side). He noticed the same applies to the number 6. Which number has Nick written in the shaded cell?

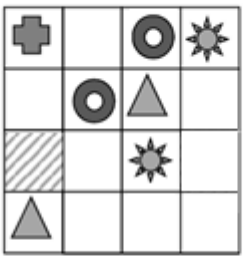


(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

7.4 Phase 2 Post- Test

If you are at least 18 years of age, you are invited to participate in this 10 minute study about math games. Although you may not personally benefit, this study is important to learn whether critical thinking skills may be influenced by mathematics games. There are no foreseeable risks, your responses will be held in confidence, and you may refuse to answer particular questions or withdraw from the study at any time. If you have no further questions and agree to participate, please continue.

1 Alice has to place different shapes in the table so that each distinct shape appears exactly once in each row and each column.



What shape should Alice place in the grey square?

(A) (B) (C) (D) (E)

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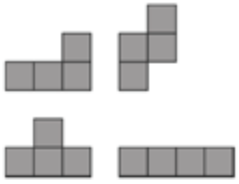
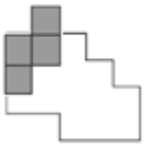
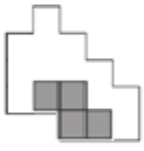
2 Ann has four pieces as shown.



With these pieces she can completely cover the shape:  .

Where should she put the piece  ?

(A) 

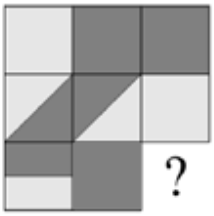
(B) 

(C) 

(D) 

(E) 

Which tile must be added to the picture so that the total light area is as large as the total dark area?



(A)  (B)  (C)  (D)  (E) 

---

4 Mary had equal numbers of white, black and striped tokens. She used some of the tokens to make the pile shown in the figure.

She still has eleven tokens which are not in the pile. How many black tokens did she have in total?

(A) 4 (B) 7 (C) 5 (D) 8 (E) 11

---

5 Nick has written each of the numbers from 1 to 9 in the cells of the 3 × 3 table. Only four of these numbers can be seen in the figure. Nick has noticed that for the number 5, the sum of the numbers in the neighbouring cells equals 13 (neighbouring cells are cells sharing a side). He noticed the same applies to the number 6. Which number has Nick written in the shaded cell?

2		3
4		1

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9